 $f(x) = a \cdot b^x$

Exponential Population Model

If a population P is changing at a constant percentage rate r each year, then

$$P(t) = P_0(1 + r)^t,$$

where P_0 is the initial population, r is expressed as a decimal, and t is time in years.

If r > 0, then P(t) is an exponential growth function.

If r < 0, then P(t) is an exponential decay function

Example 1) Tell whether the function is an exponential growth or exponential decay function, and find the constant percentage rate of growth or decay.

a) $P(t) = 78{,}963 \bullet 0.968^t$

I.V. — b = decay factor

· Exp. decay $0.968 < 1$

$1 + r = 0.968$

$-1 \quad -1$

$r = -0.032 < 0$

decay rate: 3.2% or −3.2%

b) $f(x) = 247 \bullet 2^x$

I.V. — Growth factor = 2

· Exp. Growth "doubling problem"

$1 + r = 2$

$-1 \quad -1$

$r = 1$

growth rate = 100% (100% increase)

Example 2) Determine the exponential function that satisfies the given conditions

a) Initial value=5, increasing at a rate of 17% per year

Growth rate → $f(x) = a \cdot b^x$

$a = 5$

Growth factor $1 + r = 1 + 0.17 = 1.17$

$f(x) = 5(\underbrace{1.17}_{\text{Base = Growth factor}})^x$

b) Initial population=28,900, decreasing at a rate of 2.6% per year

$P(t) = P_0(1+r)^t$

decay rate 2.6% → $r = -0.026$

decay factor $1 - 0.026 = 0.974$

$\boxed{P(t) = 28900(0.974)^t}$

Example 3) $f(x) = a \cdot b^x$

a) Find the ratio of output values that correspond to increases of 1 in the input value in order to determine the growth or decay factor
b) Determine the percent change
c) Identify or determine the value of the function when x=0 (initial value)
d) Use the information in parts (a) through (c) to define a function formula for the relationship.

d) $f(x) = 16\left(\frac{1}{4}\right)^x$

+1

x	0	1	2	3
f(x)	16	4	1	0.25

$\times \frac{1}{4}$ $\quad \times \frac{1}{4}$ $\quad \times \frac{1}{4}$

b) -75%
decr 75%
75% decrease

$\frac{4}{16} = \frac{1}{4} = \frac{0.25}{1}$ $\quad b = \frac{1}{4}$ $\quad .4$ $\quad a = 16$

+1 $\quad$ +1

x	0	1	2	3	8
g(x)	226.087	260	299	343.85	691.605

$\times 1.15$ $\quad \times 1.15$ $\quad \div 1.15$

b) 15% increase

$\frac{299}{260} = 1.15$ $\quad \frac{343.85}{299} = 1.15$ $\quad b = 1.15$

$g(x) = 226.087(1.15)^x$ $\quad a = 226.087$

Example 4) Let $f(x) = 34(1.19)^x$

A) What does the 34 represent?

Initial value

B) What does the 1.19 represent?

growth factor

C) Fill in the blank: Whenever x increases by 1, the new output value is 119 % of the old output value

D) What is the percent change?

19% increase

Example 5) Let $f(x) = 1.578(0.68)^x$

A) What does the 1.578 represent?

initial value

B) What does the 0.68 represent?

decay factor

C) Fill in the blank: Whenever x increases by 1, the new output value is 68 % of the old output value

D) What is the percent change?

-32% or 32% decrease